647510 **MSC3MatC304x** **Seat No :_____**

M.Sc.(Maths.) Semester - 3 (*CBCS*) Examination
Nov/Dec-2023 (NEW COURSE)
Discrete Mathematics (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. Figures to the right indicate marks.
2. All questions are compulsory.

Que.1(A) Answer the following question. (04)

1) State and prove Fundamental Theorem of Homomorphism of groups.

Que.1(B) Answer any TWO of the following questions. (10)

1) Show that the set of positive integers (excluding zero) with multiplication operation is a monoid.
2) Show that homomorphic image of an abelian semigroup is abelian.
3) Prove that every monoid has unique identity and unique inverse.

Que.2(A) Answer the following question. (04)

1) Let $(L, \leq)$ be a lattice and $a, b \in L$. Prove:
   (i) $a \vee b = b \iff a \leq b$
   (ii) $a \wedge b = a \iff a \leq b$

Que.2(B) Answer any TWO of the following questions. (10)

1) Show that a complement of every element in a Boolean Algebra is unique.
2) Let $(L, \leq)$ be a lattice. Suppose that $(L, \leq)$ is modular. Prove that $(L, \leq)$ is distributive iff L does not contain any sublattice which is isomorphic to the pentagon lattice.
3) Define a Boolean function. Find the values of α (0,0,1) for the Boolean expression $\alpha\,(x, y, z) = \sum(1,2,4,6)$.

Que.3(A) Answer the following question. (04)

1) State and prove Pumping Lemma.

Que.3(B) Answer any TWO of the following questions. (10)

1) What is language? Write down operations on languages.
2) Explain Moore Machine.
3) Design a finite-state automaton that accepts precisely those strings over $\{a, b\}$ that contains an odd number of a's.

Que.4(A) Answer the following question. (04)

1) Define Quantifiers. State and explain the rules of inference.

Que.4(B) Answer any TWO of the following questions. (10)

1) Let p and q be two statements, then prove that:
   (i) $\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$
   (ii) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
2) Explain conjunction and disjunction with example and truth table.
3) Define tautology. Prove that the statement Let p and q be two statements, then prove that: $p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$.

Que.5(A) Answer the following question. (04)

1) Write a short note on Hamming matrices.

Que.5(B) Answer any TWO of the following questions. (10)

1) Explain Hamming Code and find Hamming code of length seven with three parity bits.
2) Explain decoding in brief.
3) Let C be the binary linear code with generator matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

Write down a parity check matrix H for C. Explain how the minimum distance of C may be deduced from H. Find $d(C)$.

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